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FORECASTING THE REMAINING USEFUL LIFE OF MECHANICAL OBJECTS BASED ON THEIR FAILURE STATISTICS

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The article is devoted to the problem of determining the residual life of mechanical objects based on the analysis of their failure statistics using a probabilistic-physical approach. The feasibility of applying the diffusion monotonic distribution (DM-distribution) as a theoretical reliability model is substantiated. This model differs from traditional strictly probabilistic distributions in that its parameters have a clear physical interpretation: the coefficient of variation of the generalized degradation process and the average rate of change of the determining parameter. Such an approach makes it possible to combine mathematical rigor with the physical essence of wear and degradation processes.

The article considers the main characteristics of residual life-mean value and gamma-percent life and provides the formulas for their calculation using the DM-distribution. The method is shown to be convenient both in cases where the dynamics of the determining parameter are known and when long-term failure statistics are available. An example is given using experimental data from fatigue tests of V-95 alloy samples, widely employed by different researchers, which confirms the adequacy of the chosen model.

The comparison of empirical and calculated estimates demonstrates their closeness, with the error not exceeding 5 %, which is a high level of accuracy for statistical estimation practice. The practical significance of the research lies in enabling effective planning of replacement intervals and preventive maintenance for long-life objects such as nuclear power plants, pipelines, bridges, and aircraft.

Key words: residual life, DM-distribution, reliability, failure statistics, prediction.

Федухін Олександр, Стрельніков Валерій, Муха Артем. Прогнозування залишкового ресурсу (напрацювання) механічних об'єктів за статистикою їх відмов

Статтю присвячено проблемі визначення залишкового ресурсу механічних об'єктів на основі аналізу статистики їх відмов із використанням імовірнісно-фізичного підходу. У роботі обґрунтовується доцільність застосування дифузійного монотонного розподілу (DM-розподілу) як теоретичної моделі надійності. Ця модель відрізняється від традиційних строго ймовірнісних розподілів тим, що її параметри мають фізичну інтерпретацію: коефіцієнт варіації узагальненого процесу деградації та середню швидкість зміни визначального параметра. Такий підхід дає змогу поєднати математичну строгість зі зрозумілою фізичною суттю процесів зношування й руйнування об'єктів.

У статті розглянуто основні характеристики залишкового ресурсу — середнє значення та гамма-відсотковий ресурс, наведено вирази для їх обчислення на основі DM-розподілу. Показано, що методика є зручною як у випадках, коли відома динаміка зміни визначального параметра, так і за наявності довгострокової статистики відмов. Наведено приклад обробки експериментальних даних із втомних випробувань, який широко використовувався різними дослідниками, що підтверджує адекватність обраної моделі. Результати порівняння емпіричних і розрахункових оцінок свідчать про їхню близькість, зокрема, похибка не перевищує 5 %, що є високим показником точності для статистичних методів. Практичне значення дослідження полягає у можливості ефективного планування термінів заміни та профілактичних заходів для об'єктів з тривалим терміном служби, як-от атомні станції, трубопроводи, мостові споруди й авіаційна техніка.

Ключові слова: залишковий ресурс, DM-розподіл, надійність, статистика відмов, прогнозування.

Introduction. The experience of operating technical systems shows that the assigned durability and service life indicators for many types of technical systems often turn out to be significantly underestimated due to inaccuracies in reliability prediction methods or because of more favorable operating conditions. This leads to the premature decommissioning of technical systems for their intended purpose and, consequently, to the inefficient use of material resources spent on the development, production, and operation of such systems. In this regard, the assessment of the expected residual operating time (resource, service life) is of great practical importance $\pi(\tau)$. That is, the operating time of objects after the moment τ , if they have not failed (i.e., have not reached the limiting state) by that time. Knowledge of the residual operating time makes it possible to ensure more efficient further operation of the objects and to plan the timing of replacements or preventive maintenance activities. It is obvious that $\pi(\tau)$ applies only to objects that have not failed (i.e., have not reached the limiting state) by the given moment in time τ .

The purpose of this article is to develop a methodology for calculating the residual life of mechanical objects using the DM-distribution and initial data in the form of failure statistics collected over a period of monitored operation.

Theoretical Foundations, Methods, and Research Techniques. Since the residual life t_{τ}

is a random variable, its main characteristic is considered to be the distribution function $F(t \mid \tau)$, which is a conditional probability distribution function that can be represented by the residual life probability density r(t) or by the conditional probability of failure-free operation $R(r \mid \tau) = 1 - F(t \mid \tau)$, commonly referred to [1] as the residual reliability function.

The following are generally accepted [2] as the main indicators of residual life:

- mean residual life $\pi(\tau)$, defined as the mathematical expectation of the residual life after operating time τ ;
- gamma-percent residual life $\pi_{\gamma}(\tau)$, defined as the operating time starting from a certain moment in time τ , during which an object that has operated without failure up to that moment will have a conditional probability of failure-free operation equal to γ

$$\frac{R[\tau + \pi_{\gamma}(\tau)]}{R(\tau)} = \gamma. \tag{1}$$

If the initial life distribution function of the products under study F(t) (or the life probability density f(t)) is known, it is possible to derive expressions for all the above-mentioned residual life characteristics.

Fig. 1 shows the possible initial operating time probability density f(t) (dashed line) and the residual life probability density r(t)

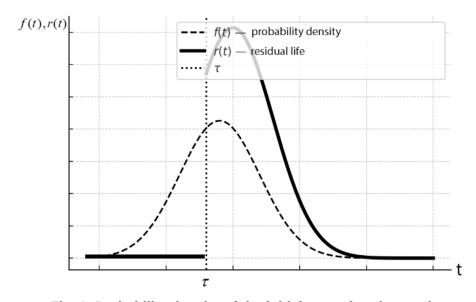


Fig. 1. Probability density of the initial operating time and probability density f(t) of the residual life, r(t) for moment τ

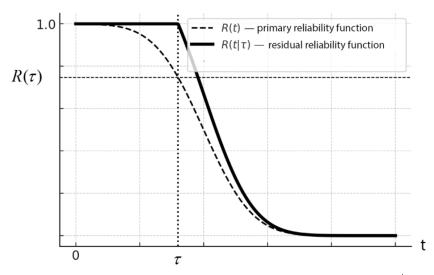


Fig. 2. Determination of the residual reliability function $R(t|\tau)$, R(t) – primary reliability function

Residual life probability density $r(t) = f(t \mid \tau)$, which is a conditional density, is obtained from the expression for the primary life probability density f(t) taking into account truncation at t the minimum τ :

$$r(t) = Cf(t), (2)$$

where
$$C = \frac{1}{\int_{0}^{\infty} f(t)dt}$$
 — normalizing factor.

Thus, the residual life probability density in the general case has the following form:

$$r(t) = \begin{cases} 0 & \text{of} \quad t < \tau, \\ \frac{f(t)}{1 - F(t)} & \text{of} \quad t \ge \tau. \end{cases}$$
 (3)

From the latter relation, the expected residual life (mathematical expectation of the residual life) is determined as follows:

$$\pi(\tau) = \int_{\tau}^{\infty} tr(t)dt - \tau = \int_{\tau}^{\infty} (t - \tau)r(t)dt = \frac{\int_{\tau}^{\infty} (t - \tau)f(t)dt}{1 - F(\tau)}.$$
 (4)

Fig. 2 shows the values of the function R(t) (bold dashed curve) and the residual reliability function $R(t \mid \tau)$ (bold solid line).

The function $R(t \mid \tau)$ corresponds to the function R(t) on the interval $[\tau; +\infty]$, with the R axis shifted to the point τ . That is, in the new system, the starting point is taken as τ (the moment of technical condition monitoring), and the value of $R(\tau)$ in the new system is taken to be equal to one.

Thus, for the function $R(t|\tau)$ on the interval $[\tau; +\infty]$, the scale of the R axis is increased by the value $\frac{1}{R(\tau)}$.

In the new scale, the section of the function R(t) on the interval $[\tau; +\infty]$ represents the residual function $R(t \mid \tau)$.

In accordance with standard [3], for technical systems in which failures of mechanical elements prevail, it is recommended to use the DM-distribution as the theoretical model of time-to-failure (to the limiting state). The diffusion DM-distribution, as a probabilistic-physical model specifically developed to describe failure distributions, is increasingly being applied, effectively replacing traditional purely probabilistic distributions such as the Weibull, lognormal, and others.

Below is the procedure for obtaining analytical expressions for the main characteristics of residual life in the case of the DM-distribution.

Let us write down the expressions for the main functions of the DM-distribution used to calculate the expected residual life $\pi(\tau)$.

DM-distribution density:

$$f(t) = \frac{(t+\mu)}{2\nu t \sqrt{2\pi\mu t}} e^{-\frac{(t-\mu)^2}{2\nu^2\mu t}},$$
 (5)

where μ is the scale parameter, which coincides with the median of the distribution, and ν is the shape parameter ν , which coincides with the coefficient of variation of the distribution.

The DM-distribution function:

$$F(t) = DM(t; \mu, \nu) = \Phi\left(\frac{t - \mu}{\nu \sqrt{\mu t}}\right), \tag{6}$$

where $\Phi(\cdot)$ – standard normal distribution function. Reliability function:

$$R(t) = 1 - DM(t; \mu, \nu) = \Phi\left(\frac{\mu - t}{\nu\sqrt{\mu t}}\right). \tag{7}$$

Residual life probability density in the case of the DM-distribution:

$$r(t) = \frac{(t+\mu)}{2\nu t \sqrt{2\pi\mu t}} \Phi\left(\frac{\mu-\tau}{\nu\sqrt{\mu t}}\right) e^{\frac{(t-\mu)^2}{2\nu^2\mu t}} \quad \text{where } t \ge \tau. \tag{8}$$

Let us write down the initial expression of the desired function $\pi(\tau)$ in accordance with (2):

$$\pi(\tau) = \frac{1}{R(\tau)} \int_{\tau}^{\infty} (t - \tau) f(t) dt, \tag{9}$$

where $R(\tau) = \Phi\left(\frac{\mu - \tau}{v\sqrt{\mu\tau}}\right)$ – probability of failure-free operation at time $t = \tau$.

Let us represent the desired integral (7) as the difference of two integrals:

$$\int_{T}^{\infty} (t - \tau) f(t) dt = \int_{T}^{\infty} t f(t) dt - \tau \int_{T}^{\infty} f(t) dt.$$
 (10)

After the corresponding substitutions and calculations, we obtain

$$\pi(\tau) = \frac{\left\{ \mu\left(1 + \frac{\nu^{2}}{2}\right) - \tau\right] \Phi\left(\frac{\mu - \tau}{\nu\sqrt{\mu\tau}}\right) + \left\{ \frac{\mu\nu^{2}}{2}\ell^{2\nu^{-2}}\Phi\left(-\frac{\mu + \tau}{\nu\sqrt{\mu\tau}}\right) + \frac{\nu\sqrt{\mu\tau}}{\sqrt{2\pi}}\ell^{\frac{(\tau - \mu)^{2}}{2\iota^{2}\mu\tau}}\right\}}{\Phi\left(\frac{\mu - \tau}{\nu\sqrt{\mu\tau}}\right)}.$$
 (11)

Let us determine the expression for the gamma-percent residual life. As is well known [2], gamma-percent residual life $\pi_{\gamma}(\tau)$ is the operating time starting from a certain moment in time τ , during which an object that has operated without failure up to that moment will have a conditional probability of failure-free operation equal to a specified level γ : $\gamma = R[\tau + \pi_{\nu}(\tau)/R(\tau)]$.

Therefore.

$$\gamma^* = \gamma \Phi\!\left(\frac{\mu - \tau}{\nu \sqrt{\mu \tau}}\right) \! = \Phi\!\left\lceil \frac{\mu - \tau - \pi_{_{\gamma}}\!\left(\tau\right)}{\nu \sqrt{\mu \left\{\tau + \pi_{_{\gamma}}\!\left(\tau\right)\right\}}}\right\rceil \! .$$

From the latter relation, we obtain:

$$\pi_{v}(\tau) = \mu(1 + U_{1-v}^{2} v^{2} / 2 - U_{1-v} v \sqrt{1 + U_{1-v}^{2} v^{2} / 4}) - \tau. (12)$$

Results and Discussion. For the purpose of evaluating the performance and adequacy of the obtained expression for $\pi(\tau)$, let us consider the experimental and predicted estimates of residual life using a specific example.

As the initial data, let us take the well-known dataset of fatigue test results for samples made of

V-95 alloy, which various authors [3–5] have used in their research.

The complete variation series and statistical characteristics (mean, coefficient of variation, etc.) are given in standard [3].

The sample size is N = 463, the sample mean is S = 169,040 cycles, and the coefficient of variation is V = 0.56. The first term of the sample variation series is $t_1 = 44,000$ cycles, and the last term is $t_{463} = 690,000$ cycles.

Based on statistical data, the following estimates of the DM-distribution parameters were obtained using the method of moments:

$$\ddot{\mu} = \frac{S(5 - V^2)}{4 + \sqrt{1 + 3V^2}} = 146 \, 127; \tag{13}$$

$$\ddot{V} = \left[\frac{2(V^2 - 1 + \sqrt{1 + 3V^2})}{5 - V^2} \right]^{1/2} = 0.56.$$
(14)

Table 1 presents the results of calculating $\pi(\tau)$ using formula (11) for ten truncation moments τ . Since the final results are known, empirical estimates of the residual life $\tilde{T}(\tau_j)$ $(j=1,\,2,\,...,\,10)$ for the corresponding moments were also calculated.

The residual life estimate $\tilde{T}(\tau_j)$ was carried out according to formula [2]:

$$\widetilde{T}(\tau) = \frac{1}{K_N(\tau)(N-k)} \sum_{i=k+1}^N Z_i,$$
(15)

where $z_i = t_i - \tau$; $K_N(\tau) = 1 - \left[1 - \tilde{P}(\tau)\right]^N$; $\tilde{P}(\tau) = 1 - \frac{k}{N}$; k – number of failed objects in the interval $(0, \tau)$.

A clearer representation of the relationships and patterns of the characteristics under study is shown in Fig. 3.

The theoretical steady-state value of the residual life (for $\tau > 2...3$ T_{cp}) based on the DM-distribution is $\pi_{\infty}(\tau) = 2\mu v^2 = 91$ 621 cycles. As can be seen, the predicted estimates of $\pi(\tau)$ are quite close to the theoretical and experimental estimates. The discrepancy between the experimental and predicted estimates of residual life for moments $\tau_1, ..., \tau_6$ (the most reliably calculated and having the greatest practical significance) averages about 5 %. This is a very high accuracy for statistical estimation practice. It should be noted that the last points (at the very tail of the distribution) have unreliable empirical estimates of $\tilde{T}(\tau)$ due to the small number of remaining non-failed samples. Thus, at time τ_{10} there were two non-failed samples, and at time τ_9 there were three samples.

Conclusions. Residual life is an important and one of the most relevant reliability characteristics

Calculated ar	nd empirica	l estimates	of residual life

τ	60000	120000	18000	240000	300000	360000	420000	480000	540000	600000
$\tilde{T}(\tau)$	114 670	92 196	94 276	93 978	93 154	82 550	75 000	74 000	80 000	55 000
π(τ)	115 229	96 268	90 665	87 112	87 168	87 395	85 802	87 428	89 836	87 419

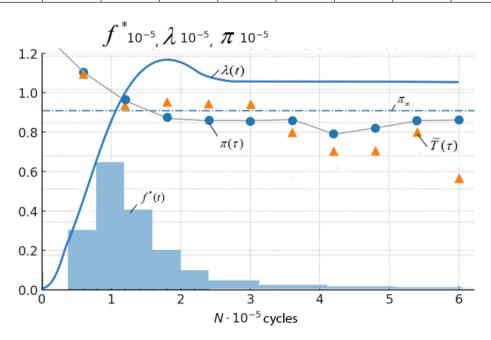


Fig. 3. Histogram $f^*(t)$, experimental residual life $\tilde{T}(\tau)$, calculated residual life $\pi(\tau)$, steady-state mean residual life π_{∞} and failure rate $\lambda(t)$

of products at any stage of their life cycle. This is especially true for objects with a long service life, particularly towards the end or after the expiration of this period. Examples of such safety-critical objects include nuclear power plants, bridges, aircraft, and similar systems.

Using the DM-distribution as a theoretical reliability model makes it possible to predict the expected residual life (operating time, service life) at any stage of storage or operation in a relatively simple and sufficiently reliable way. Currently, the most developed approaches to residual life assessment are based on analyzing the dynamics of changes in the determining life parameter

[6–19], where examples are given of residual life evaluation using the DM-distribution for a wide variety of mechanical objects.

The approach proposed in this article for assessing the residual life of mechanical objects based on failure statistics makes it possible to determine both the mean and the gammapercent residual life of mechanical objects within the framework of the DM-distribution hypothesis for time-to-failure. The prediction results obtained show good agreement with the results derived from model examples and from monitored operation of certain real mechanical objects.

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